REMARKS ON THE PROBLEM OF THE DELTA-SHAPED WING

(ZAMECHANIIA K ZADACHE O DEL'TAVIDNOM KRYLE)
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We consider a triangular plate placed at angle of attack in a uniform supersonic stream of inviscid gas under conditions such that its edges are supersonic. The problem of determining the parameters of the gas in the flow about the plate on the basis of non-linear theory we will call the problem of the delta wing. Although this problem (in particular, for the side of the wing where the stream is expanding) has received much attention [1 to 14], up to the present time a completely satisfactory solution of the problem of the delta wing has not been obtained, the authors of the present paper, therefore, have expressed differing opinions on the formulation of the problem. As a result of comments made by the first author in [7] regarding the work of the second author [8], a discussion arose between them, as a consequence of which the authors would like to make the following remarks regarding the flow pattern on that side of a triangular plate where the stream expands.

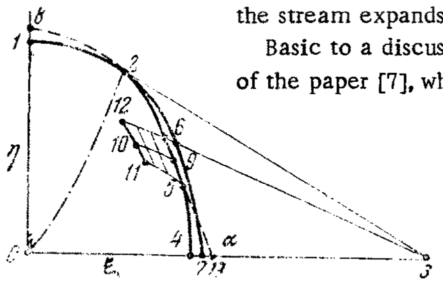

Fig. 1

Basic to a discussion of the formulation of the problem is Fig. 2 of the paper [7], which is reproduced here as Fig. 1. If we assume that shock waves do not arise in the stream, then the region of general conical flow in the central part of the wing 0-1-2-6-5-4-0 is bounded by the Mach cone 1-2, the curvilinear characteristic 2-6 of a Prandtl-Meyer flow that springs from the leading edge of the plate, part of the straight characteristic 6-5 and, for small angles of attack, the arc 5-4 of the Mach cone for the uniform flow that follows the Prandtl-Meyer flow and is adjacent to the surface of the plate. The following boundary-value problem is obtained for the determination of the conical potential F in the region 0-1-2-6-5-4-0: it is required to find the solution of Equation (1) of the paper [7], $L(F) = AF_{\xi\xi} + 2BF_{\xi\eta} + CF_{\eta\eta} = 0$, in the class of functions possessing piecewise continuous second derivatives with respect to ξ and η, with the boundary conditions that on the segment 0-1 and 0-4 respectively $F_\xi = 0$ and $F_\eta = 0$; on the arcs 1-2, 2-6, 6-5 and 5-4 are given F, F_ξ and F_η, satisfying strip conditions and conditions that are fulfilled on characteristics of Equation $L(F) = 0$. It is known beforehand that Equation $L(F) = 0$ changes its type from elliptic in that part of the region containing the point 0 to hyperbolic in the vicinity of the boundary 2-6-5. Furthermore, $AC - B^2 = 0$ on the arcs 1-2 and 4-5.

At the present time theorems of uniqueness and existence for problems of this kind do not exist, and to obtain them is very difficult. Some opinions regarding the given problem can be formed by analogy with the problem of flow of a subsonic gas stream past a symmetric profile at zero angle of attack with the formation of local supersonic zones. It is known that continuous flow of this sort around an arbitrary profile does not exist, and that, as a rule, shock waves appear in local supersonic zones. It is also known that for a given Mach number of the free stream we can select a profile about which the flow is continuous. If we compare the problem of determining the velocity potential φ for the flow about one half of the profile (adding the boundary condition $\partial\varphi/\partial n = 0$ on the axis of symmetry of the flow) and the problem of determining F in the region 0-1-2-6-5-4-0, the role of the local supersonic zone is here played by the part of the region adjacent to the arc 2-6-5. In both problems the boundary conditions are given on a "closed" contour and have an arbitrariness in one function (in the sense that if in addition to the boundary conditions one function is given, related to the solution, then the solution is completely determined in the vicinity of the given boundary). Since the boundary 1-2 is determined by the undisturbed stream, and the boundary 2-6-5-4 (Fig. 1) is completely determined by the flow in region 2-3-4-5-6, the latter plays in the problem of determining F in the region 0-1-2-6-5-4-0 the role of a profile of given shape in the plane problem, and therefore one can expect that the solution of the formulated problem does not exist, that is, shock waves appear in the flow. With regard to the location of the interior shock wave, some ideas can also be given by analogy with the plane problem. In one of the variants [7] the shock wave begins at point 2 and extends to the surface of the wing at point 7 (Fig. 1). In this case the problem of determining F in the region 0-1-2-7-0 corresponds to the problem of determining φ for a profile whose surface is unknown beforhand, and is determined so as to provide continuous flow in the local supersonic zone. In another variant [7 and 12] the interior shock wave is partly located inside the region 0-1-2-6-5-4-0, the transition of the shock wave inside this region taking place at some point on the characteristic 6-5. This case corresponds to flow past a profile with a shock wave in the local supersonic zone.

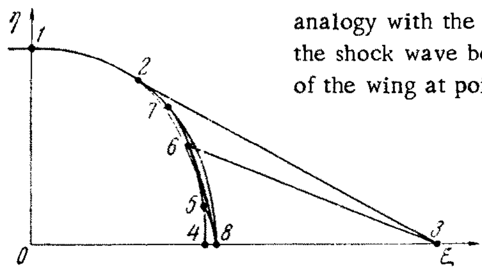

Fig. 2

The existence of an interior shock wave is indicated by asymptotic theories (11 and 14) and also by experiments [2] and others. On the basis of nonlinear theory the occurrence of an internal shock wave even at small angles of attack was predicted in references [3, 4 and 7 to 10], but there were differences of opinion as to its location and shape. Arguments given by the second author [8 to 10] were based upon study of the hodograph of the flow. Through study of the local properties of the hodograph it was established that if a solution exists corresponding to continuous flow (over the side of the wing where the stream is expanding), such a solution contains singularities of limiting lines, which follows from consideration of examples of waves. The scheme of such a solution is given in Fig. 2, and further details of the argument leading to such a representation are given in [8 to 10]. Briefly, the wave system may be represented as governing, on one side, a conical limiting line, which degenerates at point 3 and produces waves of rarefaction which, after reflection at the conical sonic line 2-8 and the conical limiting line 7-8, finally are absorbed as condensation waves by the conical limiting line 7-8 on one sheet, and, on the other side, a conical sonic line 5-4 generating condensation waves are also absorbed by the conical limiting line 7-8, but on the other sheet. This scheme of solution involves certain assumptions, and it is not certain that such a solution, including limiting lines, exists.

One objection was raised by the first author who, in the opinion of the second author, correctly observed that if one assumes, as was done in [8], that in the vicinity of point 2 the flow expands immediately downstream of the characteristic 2-6, then it can be shown that there are no discontinuities of the second derivatives of F along the characteristic 2-6, so that a point of the limiting line 7-8 cannot lie on the characteristic 2-6. This difficulty can be surmounted, however, by assuming that the flow in the vicinity of point 2 immediately downstream of the characteristic 2-6 is a compressive flow. A more serious objection is connected with the region of the simple conical wave which should be joined to the characteristic 6-5 since the latter is a straight line. As was shown by the first author in [3], a simple conical wave cannot terminate in a conical sonic line, and the curvilinear characteristics should meet in one point, as shown in Fig. 2. Such a point (point 8 in Fig. 2) must have a very complicated construction, and so far it has not been possible to construct such points. Because of the difficulty described above (and also being influenced by qualitative considerations) the first author [3, 4 and 7] drew the conclusion of the impossibility of continuous flow on the side of a triangular wing where the flow expands. Apparently a solution with limiting lines does not exist for those reasons.

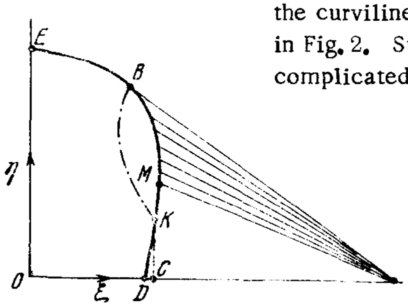

Fig. 3

If the existence of an interior shock wave now leaves no doubt, the question of its location and shape requires further investigation. The paper [13] gives a numerical solution of the problem under consideration by an iteration method, with the initial position of the interior shock wave taken downstream of the boundary 2-6-5-4 (Fig. 1). In the process of iteration the location of the shock wave was refined, and its final position is given by the curve KC in Fig. 3 (Fig. 5 in [13]). Part of the boundary of the region $OEBMKC$ consists of the rectilinear characteristic MK, which is a defect in the solution, since in this case there arises the difficulty pointed out above for simple waves. Whether this defect is a consequence of not calculating the discontinuity in acceleration propagating along the characteristics issuing from point M into the region $OEBMKC$, or a different flow pattern is realized, is not altogether clear at present. Significant results in this direction were obtained in [12], where the assumption was made that the interior shock wave penetrates into the region of generalized conical flow, and experimental data were given supporting this hypothesis. Thus in the future it is necessary to elucidate exactly which scheme is realized in actuality, and under which conditions, and obtain the corresponding unexceptionable numerical solution of the problem.

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